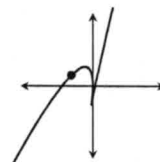


Name: Solutions.

Section: _____

The position of a certain particle is given by

$$c(t) = \left(\frac{1}{3}t^3 - t^2, t^3 - 12t \right)$$



1. Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$

$$x(t) = \frac{1}{3}t^3 - t^2 \Rightarrow \frac{dx}{dt} = t^2 - 2t = t(t-2)$$

$$y(t) = t^3 - 12t \Rightarrow \frac{dy}{dt} = 3t^2 - 12 = 3(t+2)(t-2)$$

2. At what time t is the particle stopped?

when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$
 $t = 0$ or $t = 2$ $t = -2$ or $t = 2$

no movement

when $t = 2$

horizontal tangent

3. At what time t is the particle moving parallel to the x-axis?

when $\frac{dx}{dt} \neq 0$ and $\frac{dy}{dt} = 0$
 $t \neq 0$ and $t \neq 2$ $t = -2$ or $t = 2$

when $t = -2$

4. At what time t is the particle moving parallel to the y-axis?

when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$
 $t = 0$ or $t = 2$ $t \neq -2, t \neq 2$

vertical tangent

when $t = 0$

Name: _____

Section: _____

6. Find the time at which the particle passes through
- $(0, -9)$
- .

two conditions:
on t

when $x=0$
and $y=-9$

solve condition 1

$$x=0 = \frac{1}{3}t^3 - t^2 = t^2\left(\frac{1}{3}t - 1\right)$$

$$\Leftrightarrow t=0 \text{ or } t=3$$

check condition 2

$$y(0) = 0^3 - 12 \cdot 0 = 0 \quad \times$$

$$y(3) = 3^3 - 12 \cdot 3 = \dots = -9 \quad \checkmark$$

answer question. $(x, y) = (0, -9)$ when $t=3$

7. Find the speed of the particle when it passes through
- $(0, -9)$
- .

$$\left. \frac{dx}{dt} \right|_{t=3} = 3^2 - 2 \cdot 3 = 3$$

$$\left. \frac{dy}{dt} \right|_{t=3} = 3 \cdot 3^2 - 12 = 15$$

$$\text{speed} = \sqrt{3^2 + 15^2}$$

$$= \sqrt{234} \text{ mi/hr.}$$

8. Find the slope of the curve at
- $(0, -9)$
- .

$$\text{slope} = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{15}{3} = 5 \text{ mi/mi}$$